Game Theory By Using Mathematics

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ABSTRACT: Game Theory describes human interaction involving conflict. Cooperation and competition. The term interpersonal decision theory is synonymous. The term reflects the fact that most essential features of this field are manifested in parlor games. This topic -level treatment covers large parts of the basic concepts and methods and sketches some fields of recent applications. The simultaneous occurrence of strategic, stochastic and dynamic phenomena. The fundamental role of epistemic aspects like knowledge and information and the impact of institutional and organizational structures make game theoretic analysis a highly complex task.

INTRODUCTION:'

Game Theory' is a mathematical concept, which deals with the formulation of the correct strategy that will enable an individual or entity (i.e., player), when confronted by a complex challenge, to succeed in addressing that challenge. It was developed based on the premise that for whatever circumstance, or for whatever 'game', there exists a strategy that will allow one player to 'win'. Any business can be considered as a game played against competitors, or even against customers. Economists have long used it as a tool for examining the actions of economic agents such as firms in a market.of economic agents such as firms in a market.

The ideas behind game theory have appeared through-out history, apparent in the bible, the Talmud, the works of Descartes and Sun Tzu, and the writings of Charles Darwin. However, some argue that the first actual study of game theory started with the work of Daniel Bernoulli, A mathematician born in 1700. Although his work, the "Bernoulli's Principles" formed the basis of jet engine production and operations, he is credited with introducing the concepts of expected utility and diminishing returns. Others argue that the first mathematical tool was presented in England in the 18th century, by Thomas Bayes, known as "Bayes' Theorem"; his work involved using probabilities as a basis for logical

conclusion. Nevertheless, the basis of modern game theory can be considered as an outgrowth of a three seminal works; a "Researches into the Mathematical Principles of the Theory of Wealth" in 1838 by Augustin Cournot, gives an intuitive explanation of what would eventually be formalized as Nash equilibrium and gives a dynamic idea of players best-response to the actions of others in the game. In 1881, Francis Y. Edgeworth expressed the idea of competitive equilibrium in a two-person economy. Finally, Emile Borel, suggested the existence of mixed strategies, or probability distributions over one's actions that

may lead to stable play. It is also widely accepted that modern analysis of game theory and its modern methodological framework began with John Von Neumann and Oskar Morgenstern book.We can say now that "Game Theory" is relatively not a new concept, having been invented by John von Neumann and Oskar Morgenstern in 1944. At that time, the mathematical framework behind the concept has not yet been fully established, limiting the concept's application to special circumstances only. Over the past 60 years, however, the framework has gradually been strengthened and solidified, with refinements ongoing until

today. Game Theory is now an important tool in any strategist's toolbox, especially when dealing with a situation that involves several entities whose decisions are influenced by what decisions they expect from other entities.

In, John von Neumann and Oskar Morgenstern conceived a ground-breaking mathematical theory of economic and social organization, based on a theory of games of

strategy. Not only would this reform economics, but the entirely new field of scientific inquiry it yielded has since been widely used to analyse a host of real-world phenomena from arms races to optimal policy choices of presidential candidates, from vaccination policy to major league baseball salary negotiations. In addition, it is today established throughout both the social sciences and a wide range of other sciences.

Game Theory can be also defined as the study of how the final outcome of a competitive situation is dictated by interactions among the people involved in the game (also referred to as 'players' or 'agents'), based on the goals and preferences of these players, and on the strategy that each player employs. A strategy is simply a predetermined 'way of play' that guides an agent as to what actions to take in response to past and expected actions from other agents (i.e., players in the game).In any game, several important elements exist, some of which are; the agent, which represents a person or an entity having their own goals and preferences. The second element, the utility (also called agent payoff) is a concept that refers to the amount of satisfaction that an agent derives from an object or an event. The Game, which is a formal description of a strategic situation, Nash equilibrium, also called strategic equilibrium, which is a list of strategies, one for each agent, which has the property that no agent can change his strategy and get a better payoff.Normally, any game G has three components: a set of players, a set of possible actions for each player, and a set of utility functions mapping action profiles into the real numbers. In this chapter, the set of players are denoted as I, where I is finite with, i = {1,2, 3……, I}. For each player i∈ I the set of possible actions that player i

can take is denoted by Ai, and A, which is denoted as the space of all action profiles is equal to:

$$A = A_1 \times A_2 \times A_3 \times \ldots \times A_I \text{----------------- (1)}$$

Finally, for each i∈ I, we have Ut: A → R, which denotes i's player utility function. Another notation to be defined before carrying on; suppose that a ∈ A is a strategy profile and i∈ I is a player; and then ai ∈ Ai denotes player i's action in ai and a-i denote the actions of the other I - 1 players. In this chapter, some famous examples of games, some important definitions used in games and classifications of games are presented. Throughout this chapter, a mathematical proof is presented to show when mixed strategy games can be valid and invalid in different scenarios.

: Examples of Games:
1.Prisoners' dilemma: In 1950, Professor Albert W. Tucker of Princeton University invented the Prisoner's Dilemma and, an imaginary scenario that is without doubt one of the most famous representations of Game Theory. In this game, two prisoners were arrested and accused of a crime; the police do not have enough evidence to convict any of them, unless at least one suspect confesses. The police keep the criminals in separate cells; thus, they are not able to communicate during the process. Eventually, each suspect is given three possible outcomes:
1. If one confesses and the other does not, the confessor will be released and the other will stay behind bars for ten years (i.e., -10);
2. If neither admits, both will be jailed for a short period of time (i.e., -2, -2); and
3. If both confess, both will be jailed for an intermediate period of time (i.e., six years in prison, -6).
The possible actions and corresponding sentences of the criminals are given in Table 1.

		2nd Criminal	
		Cooperate	Defect
1st Criminal	Cooperate	-2, -2	-10, 0
	Defect	0, -10	-6, -6

Table 1. Prisoners'' Dilemma game.

To solve this game, we must find the dominating strategy of each player, which is the best response of each player regardless of what the other player will play. From player one's point of

view, if player two cooperates (i.e., not admitting), then he is better off with the defect (i.e., blaming his partner). If player two defects, then he will choose defect as well. The same will work with

player two. In the end, both prisoners conclude that the best decision is to defect, and are both sent to intermediate imprisonment.

2. Battle of the sexes: Another well know game is the battle of the sexes, in which two couple argues where to spend the night out. In this example, she would rather attend an audition of Swan Lake in the opera and he would rather a football match. However, none of them would prefer to spend the night alone. The possible actions and corresponding sentences of the couple are given in Table 2.

		Female	
		Ballet	*Football*
Male	*Ballet*	2, 4	0, 0
	Football	0, 0	4, 2

Table 2. Battle of the Sexes game.

It is easy to see that both of them will either decide to go to the ballet or to the football match, as they are much better off spending the evening alone.

3.Nash Equilibrium: If a set of strategies has the property that no player can increase her payoff by changing her strategy while the other players keep their strategies the same, then the game is in Nash Equilibrium. The Nash Equilibrium is represented by these strategies and their outcomes. IN a more formal sense, a Nash equilibrium is a strategy profile in which $a_i \in A_i$,
$U (a_i, a_{-i}) \geq U(\breve{a}_i, a_{-i})$
Where $\breve{a}$, denotes another action for the player i's [1-3]. We can simply see that the action profile (defect, defect) is the Nash Equilibrium in the prisoner's dilemma game and the actions profile (ballet, ballet) and (football, football) are the ones for the battle of the sexes game.

4.Pareto efficiency: Pareto efficiency is another important concept of game theory. This term is named after Vilfredo Pareto, an Italian economist, who used this concept in his studies and defined it as; "A situation is said to be Pareto efficient if there is no way to rearrange things to make at least one person better off without making anyone worse off". More Formally, an action profile $a \in A$ is said to be Pareto if there is no action profile $a \in A$ such that for all i,
$U (a_i) \geq U (\breve{a}_i)$
In another word, an action profile is said to be Pareto efficient if and only if it is impossible to improve the utility of any player without harming another player.
In order to see the importance of Pareto efficiency, assume that someone was walking along the shore on an isolated beach finds a £20 bill on the sand. If bill is picked up and kept, then that person is better off and no one else is harmed. Leaving the bill on the sand to be washed out would be an unwise decision. However, someone might argue the fact that the original owner of the bill is worse off. This is not true, because once the owner loses the bill, he is defiantly worse off. On the other hand, once the bill is gone, he will be the same whether someone found it or it was washed out to the sea. This will lead us to another argument; assume there are two people walking on the beach and they saw the bill on the sand. Whether one of them will pick up the bill and the other will not get anything or they decide to split the bill between themselves. Who gains from finding the bill is quite different in those scenarios but they all avoid the inefficiency of leaving it sitting on the beach.

5.Pure and mixed strategy Nash Equilibrium:
In any game someone will find pure and mixed strategies, a pure strategy has a probability of one, and will be always played. On the other hand, a mixed strategy has multiple purse strategies with probabilities connected to them. A player would only use a mixed strategy when she is indifferent between several pure strategies, and when keeping the challenger guessing is desirable, that is when the opponent can benefit from knowing the next move. Another reason why a player might decide to play a mixed strategy is when a pure strategy is not dominated by other pure strategies, but dominated by a mixed strategy. Finally, in a game without a pure strategy Nash Equilibrium, a mixed strategy may result in a Nash Equilibrium.
From the battle of the sexes game, we can see the mixed strategy Nash equilibria are the action profile (ballet, ballet) and (football,

football). In order to drive that, we will assume first that the women will go to the ballet and the man will play some mixed strategy σ. Then the utility of playing this action will be
$U_f = f(\sigma)$

Then, $U_B = \sigma_B(4) + (1-\sigma_B)(0)$ therefore in another word, the women get '4' some percentage of the time and '0' for the rest of the time. Assuming the women will be going with herUF = $\sigma_B(0) + (1 - \sigma_B)(2)$ she will get '0' some percentage of the time and '2' for the rest of the time. Setting the two equations equal to each other and solving for σ, this will $\sigma_B = 1/3$. This means that in this mixed strategy Nash equilibrium, the

man is going to the ballet third of the time and to going to the football match two-third of the time. Taking another look to the Table 2-2, we can see that the game is symmetrical against the strategies, which means that the women will decide to go the ballet two-third of the time and third of the time to go to the football match. In order to calculate the utility of each player in this game, we need to multiply the probability distribution of each action with by the user strategy, as shown in Table 3. We can simply see that the utility of both players is '4/3', which means that if they won't communicate with each other to decide where to go, they are both better-off to use mix strategies.

Male	Female	Ballet (2/3)		Football (1/3)	
Ballet (1/3)		2/9	2, 4	1/9	0, 0
Football (2/3)		4/9	0, 0	2/9	4, 2

Pure and Mixed Strategies, Battle of the Sexes example.

5.Valid and invalid mixed strategy Nash Equilibrium:
This section shows how mixed strategies can be invalid with games in general forms.

Recalling the prisoner's dilemma game from the previous section, where we going to solve the general class of the game by removing the numbers from the table and use the following variables;

1st Criminal	2nd Criminal	Cooperate	Defect
Cooperate		B, b	D, a
Defect		A, d	C, c

Valid and Invalid Mixed Strategy Nash Equilibrium, Prisoners' Dilemma example.

Where we have, A > B > C > D and a > b > c > d. We will simply start to solve this game the same way we did before, we will start looking for the dominate strategies. From the player one point of view, if player two cooperate then player one will not as A > D. If player two defect, then player one will defect as well as C > D. Doing the same thing for player two; if player one confesses, then player two will defect as a > d. If player one defect, then player two will defect as well as c > d. Then, the only sensible equilibrium will be (Don't confess, don't confess).

To make sure that there is no mixed strategy Nash equilibrium in this scenario, we need to find the utility of player two confessing as a function of some mixed strategy of player one. That is, some percentage of the time player two will get b and for the rest of the time will get d.

Mathematically this will be; $U_C = \sigma_C(b) + (1 - \sigma_C)(d)$. Then, we do the same to find what theutility of player two will be as function of player one mixed strategy. This can be shown as; $U_D = \sigma_C(a) + (1 - \sigma_C)(c)$. To find the mixed strategy, U_C must be equal to U_D, and that will lead us to the following equation;

$$\sigma_c = \frac{c-d}{b-d-a+c}$$

In order to proof that this is a valid mixed strategy Nash equilibrium, the following condition must be satisfied; $Pr(i) \in [0,1]$ (i.e., no event can occur with negative probability and no event can occur with probability greater than one). That is the probability that this strategy will happen is greater than zero and not less than one. For the first case, when $\sigma_C \geq 0$, the nominator and the denominator must be both positive or negative, otherwise, this

mixed strategy will be invalid. Recalling our assumption, $a > b > c > dm$ then the nominator must be greater than zero, the denominator must be greater than zero as well. That is $b + c - a - d > 0$, which can be re-arranged as $b + c > a + d$, at this point we can be sure whether this will give us the right answer of whether this is a valid mixed strategy or not as there will be some times where $b + c$ is greater than $a + d$ and some times where it is not. So, for the mixed strategy Nash equilibrium for

this game does exist, σ_C must be less than or equal to one. This will lead us to the following equation:

$$\frac{c - d}{b - d - a + c} \leq 1$$

That is $c - d \leq b - d - a + c$, which can be solved to $a \leq b$, which is not right as this violate or rule that a $> b$, so this is an invalid mixed strategy. Thus, we proved that there is no mixed strategy Nash equilibrium in this game and the two players will defect.

Valid and Invalid Mixed Strategy Nash Equilibrium, Battle of the Sexes example.

On the other hand, if we work for the example of the Battle of the Sexes game. Table shows the game in general format, were we removed the numbers again and used the following variables; $A \geq B \geq C \geq 0$ and $a \geq b \geq c \geq 0$. Following the same procedure we used in the previous example, we can solve for the man mixed strategy when his partner goes to watch the match, which will lead us to the following equality: $UF = \sigma F(b) + (1 - \sigma_F)(c)$, as the women get b some percentage of the time and get c the rest of the time. If she decides to go to the ballet, the equality becomes; $UB = \sigma F(c) + (1 - \sigma_F)(a)$. Now, taking these two equations to solve for the man mixed strategy, we can finally get: $\sigma_F = (a - c)/(a + b - 2c)$. In order to prove that this mixed strategy is valid, the same condition used before must be satisfied, $Pr(i) \in [0,1]$. That is, $\sigma_F \geq 0$, we already have $a > c$, then the numerator is positive and greater than zero. For the denominator to be positive, $(a + b - 2c)$ must be positive. That is $a + b - 2c \geq 0$, which can be arranged as $a - c \geq c - b$, which proves that the denominator is positive as this is always true.

We must prove that $\sigma_F \leq 1$ to prove the validity of such mixed strategy. That means we must prove the following; $a - c \leq a + b - 2c$, which can be arranged to the following $c \leq b$, which is true as we already mentioned that $b \geq c \geq 0$. Thus, we have proved that there exist three equilibriums in this game, the two players can go the Ballet or to the match together or each one of them can go to

their preferred show with a probability of $(a - c)/(a + b - 2c)$.

Classification of game theory:

Games can be classified into different categories according to certain significant features. The terminology used in game theory is inconsistent, thus different terms can be used for the same concept in different sources. A game can be classified according to the number of players in the game, it can be designated as a one-player game, two-player game or nplayers game (where n is greater than '2'). In addition, a player need not be an individual person; it may be a nation, a corporation, or a team comprising many people with shared interests.

Non-cooperative and cooperative (coalition) games:

A game is called non-cooperative when each agent (player) in the game, who acts in her self-interest, is the unit of the analysis. While the cooperative (Coalition) game treats groups or subgroups of players as the unit of analysis and assumes that they can achieve certain payoffs among themselves through necessary cooperative agreements. In non-cooperative games, the actions of each individual player are considered and each player is assumed to be selfish, looking to improve its own payoff and not taken into account others involved in the game. So, non-cooperative game theory studies the strategic choices resulting from the interactions among competing players, where each player chooses its strategy independently for

improving its own performance (utility) or reducing its losses (costs). On the other hand, Cooperative game theory was developed as a tool for assessing the allocation of costs or benefits in a situation where the individual or group contribution depends on other agents' actions in the game. The main branch of cooperative games describes the formation of cooperating groups of players, referred to as coalitions, which can strengthen the players' positions in a game. In Telecommunications systems, most game theoretic research has been conducted using non-cooperative games, but there are also approaches using coalition games. Studying the selfishness level of wireless node in heterogeneous ad-hoc networks is one of the applications of coalition games. It may be beneficial to exclude the very selfish nodes from the network if the remaining nodes get better QoS that way.

Strategic and extensive games:
One way of presenting a game is called the strategic, sometimes called static or normal, form. In this form the players make their own decisions simultaneously at the beginning of the game, the players have no information about the actions of the other players in the game. The prisoner's dilemma and the battle of the sexes are both strategic games. Alternatively, if players have some information about the choices of other players, the game is usually presented in extensive, sometimes called as a game tree, form. In this case, the players can make decisions during the game and they can react to other players' actions. 8 Game Theory Such form of games can be finite (one-shot) games or infinite (repeated) games. In repeated games, the game is played several times and the players can observe the actions and payoffs of the previous game before proceeding to the next stage.

Zero-sum games:
Another way to categorize games is according to their payoff structure. Generally speaking, a game is called zero-sum game (sometimes called if one gains, another losses game, or strictly competitive games) if the player's gain or loss is exactly balanced those of other players in the game. For example, if two are playing chess, one person will lose (with payoff '-1') and the other will win (with payoff'+1'). The win added to the loss equals zero. Given that sometimes a loss can be a gain, real life examples of zero-sum game can be very difficult to find. Going back to the chess example, a loser in such game may gain as much from his losses as he would gain if he won. The player may become better player and gain experience as a result of

losing at the first place. In telecommunications systems, it is quite hard to describe a scenario as a zero-sum game. However, in a bandwidth usage scenario of a single link, the game may be described as a zero-sum game.

Games with perfect and imperfect information:
A game is said to be a perfect information game if each player, when it is her turn to choose an action, knows exactly all the previous decisions of other players in the game. Then again, if a player has no information about other players' actions when it is her turn to decide, this game is called imperfect information game. As it is hardly ever any user of a network knows the exact actions of the other users in the network, the imperfect information game is a very good framework in telecommunications systems. Nevertheless, assuming a perfect information game in such scenarios is more suitable to deal with.

Games with complete and incomplete information: In games with "complete information", all factors of the game are common knowledge to all players. That is, each individual player is fully aware of other players in the game, their strategies and decisions and the payoff of each player. As a result, a complete information game can be represented as an efficient perfectly competitive game. On the other hand, in the "incomplete information" games, the player's dose not has all the information about other players in the game, which made them not able to predict the effect of their actions on others. One of the very well-known types of such games is the sealed-bid auctions, in which a player knows his own valuation of the good but does not know the other bidders' valuation. A combination of incomplete but perfect information game can exist in a chess game, if one player knows that the other player will be paid some amount of money if a particular event happened, but the first player does not know what the event is. They both know the actions of each other, perfect information game, but does not know the payoff function of the other player, incomplete information game.

Rationality in games: the most fundamental assumption in game theory is rationality. It implies that every player is motivated by increasing his own payoff, i.e., every player is looking to maximizehis own utility. John V. Neumann and Morgenstern justified the idea of maximizing the expected payoff in their work in 1944. However, previous studies have shown that humans do not always act rationally. In fact, humans use a

propositional calculus in reasoning; the propositional calculus concerns truth functions of propositions, which are logical truths (statements that are true in virtue of their form). For this reason, the assumption of rational behaviour of players in telecommunications systems is more justified, as the players are usually devices programmed to operate in certain ways.

Evolutionary games: Evolutionary game theory started its development slightly after other games have been developed. This type of game was originated by John Maynard Smith formalization of evolutionary stable strategies as an application of the mathematical theory of games in the context of biology in 1973. The objective of evolutionary games is to apply the concepts of non-cooperative games to explain such phenomena which are often thought to be the result of cooperation or human design, for example; market information, social rules of conduct and money and credit. Recently, this type of games has become of increased interest to scientist of different background, economists, sociologists, anthropologists and also philosophers. One of the main reasons behind the interest among social scientists in the evolutionary games rather than the traditional games is that the rationality assumptions underlying evolutionary game theory are, in many cases, more appropriate for the modelling of social systems than those assumptions underlying the traditional theory of games.

Applications of game theory:in telecommunications Communications systems are often built around standard, mostly open ones, such as the TCP/IP (Transmission Control Protocol/Internet Protocol) standard in which the internet is based. Devices that we use to access these systems are being designed and built by a diversity of different manufactures. In many cases, these manufacturers may have an incentive to develop products, which behave "selfishly" by seeking a performance advantage over other network users at the cost of overall network performance. On the other hand, end users may have the ability to force these devices in order to work in a selfish manner. Generally speaking, the maximizing of a player's payoff is often referred to as selfishness in a game. This is true in the sense that all the players try to gain the highest possible utility of their actions. However, a player gaining a high utility does not necessarily mean that the player acts selfishly. As a result, systems that are prepared to cope with users who behave selfishly need to be designed. If the designs of such systems are possible, designers should make sure that

selfish behaviour within the system is unprofitable for individuals. When designing such system is not possible, they should be at least aware of the impact of such behaviour on the operation of the specified system. One important thrust in these efforts focuses on designing high-level protocols that prevent users from misbehaving and/or provide incentives for cooperation. To prevent misbehaviour, several protocols based on reputation propagation have been proposed in the literature, e.g... The mainstream of existing research in telecommunications networks focused on using non-cooperative games in various applications such as distributed resource allocation, congestion control, power control, and spectrum sharing in cognitive radio, among others. This need for non-cooperative games led to numerous tutorials and books outlining its concepts and usage in communication, such as, another thrust of research analyses the impact of user selfishness from a game theoretic perspective, e.g., Since the problem is typically too involved, several simplifications to the network model are usually made to facilitate analysis and allow for extracting insights. For example, in the wireless nodes are assumed to be interested in maximizing energy efficiency. At each time slot, a certain number of nodes are randomly chosen and assigned to serve as relay nodes on the source- destination route. The authors derive a Pareto optimal operating point and show that a certain variant of the well-known TIT-FOR-TAT algorithm converges to this point. In the authors assume that the transmission of each packet costs the same energy and each session uses the same number of relay nodes. Another example which studies the Nash equilibrium of packet forwarding in a static network by taking the network topology into consideration. More specifically, the authors assume that the transmitter/receiver pairs in the network are always fixed and derive the equilibrium conditions for both cooperative and non-cooperative strategies. Similar to, the cost of transmitting each packet is assumed fixed. It is worth noting that most, if not all of, the works in this thrust utilize the repeated game formulation, where cooperation among users is sustainable by credible punishment for deviating from the cooperation point.Cooperative games have also been widely explored in different disciplines such as economics or political science. Recently, cooperation has emerged as a new networking concept that has a dramatic effect of improving the performance from the physical layer up to the networking layers. However, implementing cooperation in large scale communication networks faces several challenges such as adequate

modelling, efficiency, complexity, and fairness, among others. In fact, several recent works have shown that user cooperation plays a fundamental role in wireless networks. From an information theoretic perspective, the idea of cooperative communications can be traced back to the relay channel. More recent works have generalized the proposed cooperation strategies and established the utility of cooperative communications in many relevant practical scenarios, such as and in another line of work, in the authors have shown that the simplest form of physical layer cooperation, namely multi hop forwarding, is an indispensable element in achieving the optimal capacity scaling law in networks with asymptotically large numbers of nodes. Multi-hop forwarding has also been shown to offer significant gains in the efficiency of energy limited wireless networks. These physical layer studies assume that each user is willing to expend energy in forwarding packets for other users. This assumption is reasonable in a network with a central controller with the ability to enforce the optimal cooperation strategy on the different wireless users. The popularity of ad-hoc networks and the increased programmability of wireless devices, however, raise serious doubts on the validity of this assumption, and hence, motivate investigations on the impact of user selfishness on the performance of wireless networks. The following chapters will be full of more details about the applications of game theory in wireless telecommunications systems, including applications of game theory in interface selections mechanisms, Mobile IPv6 protocol extensions, resource allocations androuting in Ad-Hoc wireless network and spectrum sharing in Cognitive Radio networks.

SYNOPSIS: This chapter gives a detailed insight in the game theory definition, classifications and applications of games in telecommunications. Prisoners Dilemma and the Battle of the Sexes games have been discussed in details, showing different strategies from the players and discussing the expected outcome of such games. Nash Equilibrium and Pareto Efficient terms are discussed in details with detailed examples. Moreover, we have discussed mixed strategies in games and mathematically proved that a mixed strategy in Prisoners' Dilemma example does not exist. We have also proved that a mixed strategy exists in the battle of the sexes game. Finally, after classifying games into different categories, an introduces to the applications of game theory in Telecommunications.

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